AYDIN ADNAN MENDERES UNIVERSITY FACULTY OF ENGINEERING

Department of Electrical and Electronics Engineering

EE454 – Digital Image Processing

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Presentation 9

Dr. Adem Ükte

Homomorphic Filtering in Frequency Domain

- The illumination-reflectance model introduced in previous sections can be used to develop a frequency domain procedure for improving the appearance of an image by simultaneous intensity range compression and contrast enhancement.

- From the discussion in that section, an image $f(x,y)$ can be expressed as the product of its illumination, $i(x,y)$, and reflectance, $r(x,y)$, components:

$$f(x,y) = i(x,y)r(x,y)$$

- This equation cannot be used directly to operate on the frequency components of illumination and reflectance because the Fourier transform of a product is not the product of the transforms:

$$\mathcal{F}[f(x,y)] \neq \mathcal{F}[i(x,y)]\mathcal{F}[r(x,y)]$$

- However, suppose that we define

$$z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$$

- Note that, If $f(x,y)$ has any zero values, a 1 must be added to the image to avoid having to deal with $\ln 0$. The 1 must be then subtracted from the final result.

- If Fourier transform of both sides of the equation is taken then,

$$\mathcal{F}[z(x,y)] = \mathcal{F}[\ln f(x,y)] = \mathcal{F}[\ln i(x,y)] + \mathcal{F}[\ln r(x,y)]$$

- or

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

- where $F_i(u,v)$ and $F_r(u,v)$ are the Fourier transforms of $\ln i(x,y)$ and $\ln r(x,y)$, respectively.

Homomorphic Filtering in Frequency Domain

- We can filter $Z(u, v)$ using a filter transfer function $H(u, v)$ so that

$$S(u, v) = H(u, v)Z(u, v) = H(u, v)F_i(u, v) + H(u, v)F_r(u, v)$$

- The filtered image in the spatial domain is then

$$s(x, y) = \mathcal{F}^{-1}[S(u, v)] = \mathcal{F}^{-1}[H(u, v)F_i(u, v)] + \mathcal{F}^{-1}[H(u, v)F_r(u, v)]$$

- By defining

$$i'(x, y) = \mathcal{F}^{-1}[H(u, v)F_i(u, v)]$$

- and

$$r'(x, y) = \mathcal{F}^{-1}[H(u, v)F_r(u, v)]$$

- we can express the filtered image in the spatial domain in the form

$$s(x, y) = i'^{(x,y)} + r'(x, y)$$

- Finally, because $z(x, y)$ was formed by taking the natural logarithm of the input image, we reverse the process by taking the exponential of the filtered result to form the output image:

$$g(x, y) = e^{s(x,y)} = e^{i'(x,y)}e^{r'(x,y)} = i_0(x, y)r_0(x, y)$$

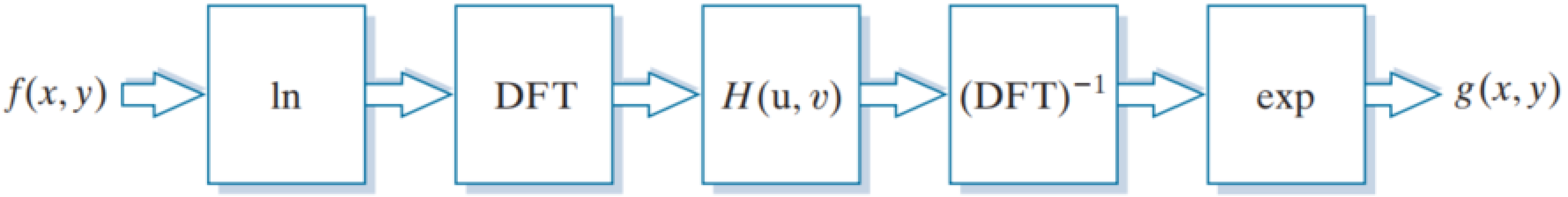
- where

$$i_0(x, y) = e^{i'(x,y)} \quad \text{and} \quad r_0(x, y) = e^{r'(x,y)}$$

- are the illumination and reflectance components of the output (processed) image.

Homomorphic Filtering in Frequency Domain

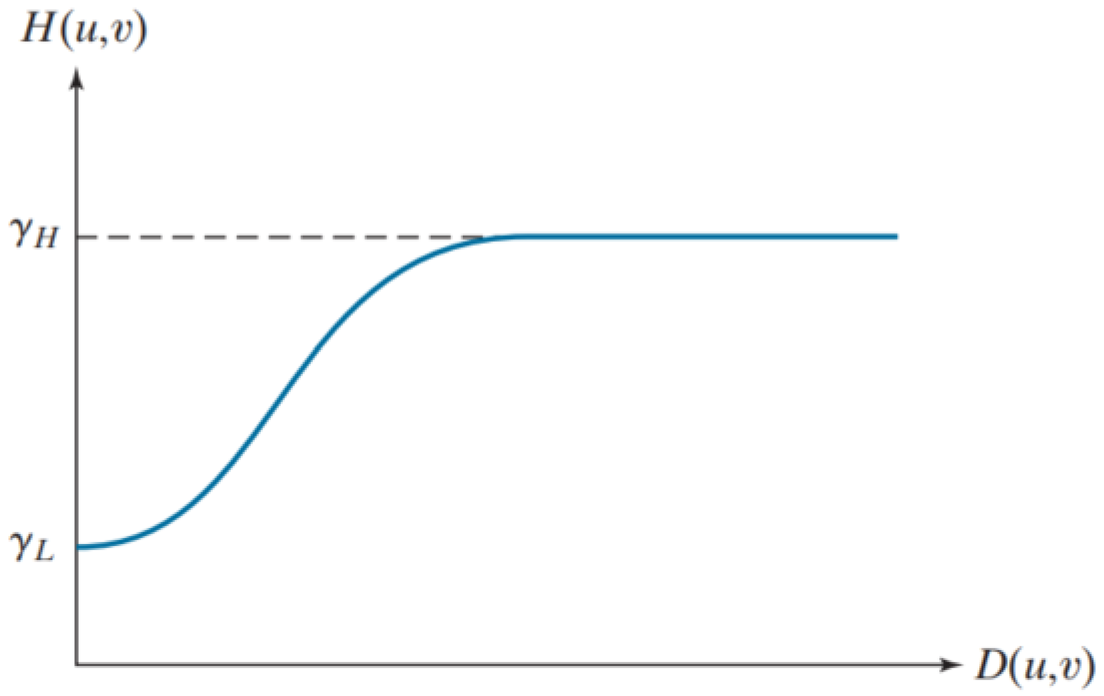
- Below figure shows a summary of the filtering approach just derived.



- This method is based on a special case of a class of systems known as homomorphic systems.
- In this particular application, the key to the approach is the separation of the illumination and reflectance components.
- The homomorphic filter transfer function, $H(u, v)$, then can operate on these components separately.
- The illumination component of an image generally is characterized by slow spatial variations, while the reflectance component tends to vary abruptly, particularly at the junctions of dissimilar objects.
- These characteristics lead to associating the low frequencies of the Fourier transform of the logarithm of an image with illumination, and the high frequencies with reflectance.
- Although these associations are rough approximations, they can be used to advantage in image filtering.

Homomorphic Filtering in Frequency Domain

- A good deal of control can be gained over the illumination and reflectance components with a homomorphic filter.
- This control requires specification of a filter transfer function $H(u,v)$ that affects the low- and high-frequency components of the Fourier transform in different, controllable ways.
- Below figure shows a cross section of such a function.

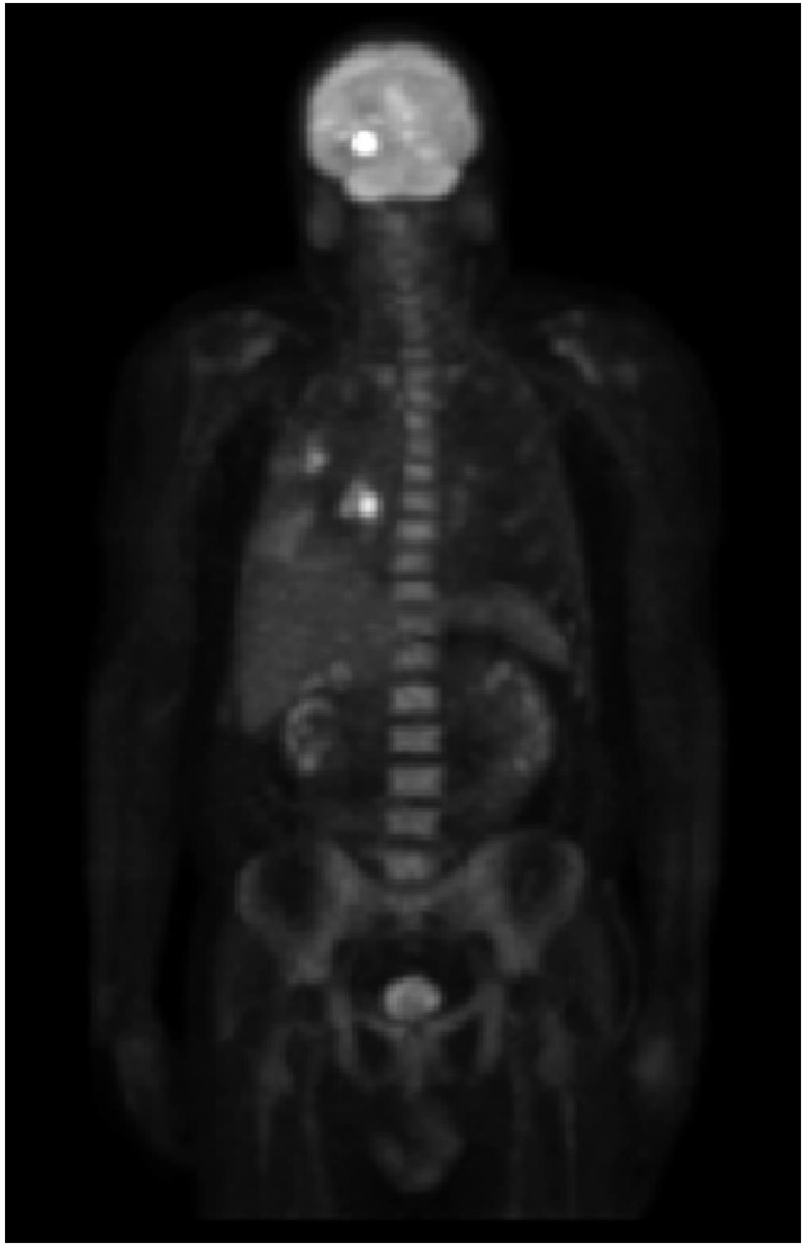


- If the parameters γ_L and γ_H are chosen so that $\gamma_L < 1$ and $\gamma_H \geq 1$, the filter function in the figure will attenuate the contribution made by the low frequencies (illumination) and amplify the contribution made by high frequencies (reflectance).
- The net result is simultaneous dynamic range compression and contrast enhancement.
- The shape of the function in the figure can be approximated using a highpass filter transfer function.
- For example, using a slightly modified form of the GHPF function yields the homomorphic function

$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-cD^2(u,v)/D_0^2}\right] + \gamma_L$$

- where $D(u,v)$ is the distance function defined in previous sections and constant c controls the sharpness of the slope of the function as it transitions between γ_L and γ_H .

Original PET image



- **Example 20:** Left figure shows full body PET (Positron Emission Tomography) scan of size 1162×746 pixels.

```
clc;clear;close all;
f=imread('PET_image.tif');
figure,imshow(f)
title('Original PET image',...
'FontSize',10,'FontName','Comic Sans MS')
```

- The image is slightly blurred and many of its low-intensity features are obscured by the high intensity of the "hot spots" dominating the dynamic range of the display.
- These hot spots were caused by a tumor in the brain and one in the lungs.
- Homomorphic filtering using the filter transfer function $H(u,v)$in previous page with $\gamma_L = 0.5$, $\gamma_H = 1.5$, $c = 0.5$, and $D_0 = 60$ pixels.

- **Example 20 (Cont.):**

```
[M,N]=size(f);
P=2*M; Q=2*N;
u=0:P-1; v=0:Q-1;
[v,u]=meshgrid(v,u);
D=sqrt((u-P/2).^2+(v-Q/2).^2);
D0=60;
Gamma_L=0.5; Gamma_H=1.5;
c=0.5;
H_HomFilt=(Gamma_H-Gamma_L)*(1-exp(-c*D.^2/D0^2))+Gamma_L;
z=log(double(f+1));
s=func_freq_filt(z,H_HomFilt);
g=exp(s); g=g-1;
g=func_cont_strch(g,0,255); % Scaling
figure,subplot(121),imshow(f),...
title('Original PET image','FontSize',10,'FontName','Comic Sans MS')
subplot(122),imshow(uint8(g)),title(['Hom. filtered image D_0=',num2str(D0),...
' \gamma_L=',num2str(Gamma_L),' \gamma_H=',num2str(Gamma_H),' c=',num2str(c)],...
'FontSize',10,'FontName','Comic Sans MS')
```

$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-cD^2(u,v)/D_0^2}\right] + \gamma_L$$

$$z(x,y) = \ln f(x,y)$$

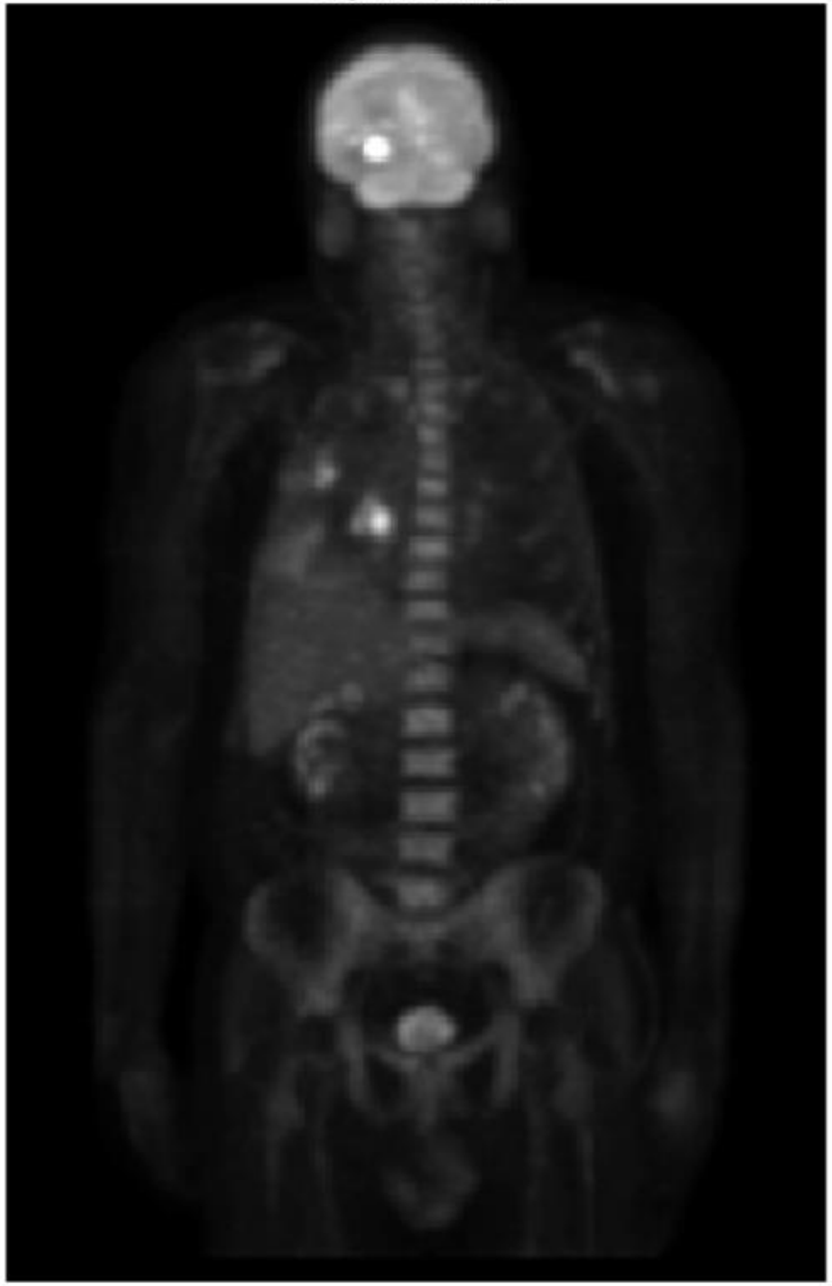
$$s(x,y) = \mathcal{F}^{-1}[H(u,v)Z(u,v)]$$

$$g(x,y) = e^{s(x,y)}$$

- **Example 20 (Cont.):**

- Note that how much sharper the hot spots, the brain, and the skeleton are in the processed image,
- and how much more detail is visible in this image, including, for example, some of the organs, and the shoulders.
- By reducing the effects of the dominant illumination components (the hot spots), it became possible for the dynamic range of the display to allow lower intensities to become more visible.
- Similarly, because the high frequencies are enhanced by homomorphic filtering, the reflectance components of the image (edge information) were sharpened considerably.
- The enhanced image is a significant improvement over the original.

Original PET image



Hom. filtered image D_0=60 γ_L=0.5 γ_H=1.5 c=0.5

END OF PRESENTATION 9

References:

✓Gonzalez, Woods, and Eddins, Digital Image Processing Using MATLAB 2nd Ed., 2009.

✓Gonzalez and Woods, Digital Image Processing 4th Ed., 2018.

✓Kizilkaya A., Görüntü İşleme Teknikleri ve Uygulamaları Ders Notları, PAÜ, 2008.